$\Delta H_{2(298)} + \int_{298}^{T_1} c_2 dT_1$; $i_2 = \Delta H_{2(298)} + \int_{298}^{T_2} c_2 dT_2$; $\Delta H_{W(298)}$, $\Delta H_{2(298)}$ are the standard enthalpies of water and the substance; c_W, c_2 are the specific heat capacities of water and the substance of the particles, respectively.

If we neglect the quantity $D_2u_\sigma^1/Dt$, then, the equation of the influx of heat to the phase interface leads to a finite algebraic equation for T_σ

$$4\pi a^2 f[\beta_1(T_1 - T_\sigma) + \beta_2(T_2 - T_\sigma)] - \rho_2^0 f\eta(i_2 - i_1) = 0.$$

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FLOW OF GENERALIZED NEWTONIAN AND BINGHAM LIQUIDS IN AN ANNULAR CAPILLARY

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The laws of motion of Newtonian [1, 2], viscoplastic [3-5], and non-Newtonian [6] liquids in an annular capillary (in the gap between two coaxial cylindrical tubes) have already been obtained. In the present paper we solve the problem of established horizontal flow of generalized Newtonian and Bingham liquids [7-9] in an annular capillary.

Let R_1 and R_2 be the internal and external radii, respectively, of the tubes forming the annular capillary, and r the radial cylindrical coordinate of a liquid particle in the flow cross section.

The flow of a generalized Newtonian liquid in a capillary under the action of a hydraulic pressure gradient I proceeds within the expanding ring $r_1 \le r \le r_2$ in such a way that the velocity v(r) at some intermediate $r = r_0$ is a maximum and. decreasing nonsymmetrically in the direction of the walls, is a minimum at $r = r_1$ and $r = r_2$. We can accordingly distinguish two flow zones with different velocity laws $v_j(r)$ in the flow cross section. In the first zone ($j = 1$, $r_1 \le r \le r_0$) the velocity gradient $dv_1(r)/dr \ge 0$ and in the second zone ($j = 2$, $r_0 \le r \le r_2$) $dv_2/dr \le 0$.

Considering the balance of the forces applied to an elementary annular layer of liquid in each zone we have

$$d\tau_j(r)/dr + \tau_j(r)/r = (-1)^j \rho g I, \tag{1}$$

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where

$$\tau_j(r) = -(-1)^j \eta dv_j(r)/dr + \eta N_j(r) \tag{2}$$

is the tangential shear stress with its limiting value $\tau_\bullet = \eta N_j(r)$ in the given layer [7]; ρ and η are the density and viscosity of the liquid; g is the gravitational acceleration.

Integrating (1) and taking into account (2) with boundary conditions

$$\text{a)}\quad v_j(r_j) = 0,\quad \text{b)}\quad \left.\frac{dv_j(r)}{dr}\right|_{r=r_j} = 0,\quad \text{c)}\quad v_1(r_0) = v_2(r_0),$$

$$\text{d)}\quad \left.\frac{dv_1(r)}{dr}\right|_{r=r_\bullet} = -\left.\frac{dv_2(r)}{dr}\right|_{r=r_\bullet} = 0,$$

we find an expression for the velocity of the liquid particles in each zone:

$$v_j(r) = -\frac{\rho g I}{4\eta}\left(r^2 - r_j^2 - 2r_0^2 \ln\frac{r}{r_j}\right) - (-1)^j\left[r_0 N_0 \ln\frac{r}{r_j} - \int\limits_{r_j}^{r} N_j(r)\,dr\right] \quad (N_0 = N_j(r_0)). \tag{3}$$

For determination of r_j and $r_\bullet$ with a known law of variation $N_j(r)$ we can use conditions b and c. In addition, condition b enables us to obtain a relation connecting the internal flow picture with the external forces:

$$I = (-1)^j (2\eta/\rho g)\,[r_j N_j(r_j) - r_0 N_0]/(r_j^2 - r_0^2). \tag{4}$$

Hence, for $r_j \to r_\bullet$ and $r_j \to R_j$ we can establish upper and lower limits of the effective gradients $I_\bullet$ and I_R at which there is no motion of the liquid in the capillary and no further expansion of the section, respectively. A further increase in I ($I > I_R$), causing a flow through almost the entire volume of the capillary, is limited by the critical value of the pressure gradient I_*, above which laminar flow is converted to turbulent flow.

The liquid flow rate Q in the annular capillary is

$$Q = 2\pi \sum_{j=1}^{2} (-1)^j \int\limits_{r_\bullet}^{r_j} r v_j(r)\,dr.$$

Substituting in this the expression $v_j(r)$ from (3) and integrating, we obtain

$$Q = \frac{kI}{R_2^4}\left[r_2^4 - r_1^4 - 4r_0^2\left(r_2^2 - r_1^2 - r_0^2 \ln\frac{r_1}{r_2}\right) + 2\pi \sum_{j=1}^{2} \int\limits_{r_0}^{r_j} r\,dr \int\limits_{r_j}^{r} N_j(x)\,dx + \right.$$
$$\left. + \frac{\pi}{2} r_0 N_0 \left(r_2^2 + r_1^2 - r_0^2 \ln\frac{r_0^2}{r_1 r_2}\right),\right. \quad k = \pi\rho g R_2^4/(8\eta). \tag{5}$$

We consider some representative cases of $N_j(r)$.

$$N_j(r) = M_j\left(\frac{r - r_0}{R_j - r_0}\right)^n,\quad n > 1;\quad N_j(r) = \alpha_j r\left(\frac{r^2 - r_0^2}{R_j^2 - r_0^2}\right)^m,$$
$$m > 0\ (N_0 = 0,\ N_j(R_j) = M_j), \tag{6}$$

where n, m, and M_j, α_j are, respectively, parameters characterizing the non-Newtonian behavior of the liquid in the volume and at the contact boundary.

Substituting the expression $N_j(r)$ from (6) in (4) and calculating the corresponding limits, we find

$$I_0 = 0,\quad I_R = (-1)^j (2\eta R_j M_j/\rho g)/(R_j^2 - r_0^2). \tag{7}$$

Here we have the relations

$$N_1(r_1) = N_2(r_2),\quad r_0^2 = s r_2^2 \ \text{ for }\ I_0 \leqslant I \leqslant I_R,$$
$$r_0^2 = \theta R_1 R_2 \ \text{ for }\ I_R \leqslant I < I_*, \tag{8}$$

where

$$s = r_1/r_2;\ \theta = (c+\mu)/(1+c\mu);\ c = R_1/R_2;\ \mu = M_1/M_2.$$

Now, introducing (6)-(8) into (5) and performing the integration under the summation sign we obtain expressions for the flow Q of a generalized Newtonian liquid in an annular capillary

$$Q = kI\begin{cases}\left(\frac{r_2}{R_2}\right)^4(1-\sqrt{s})^2(1-s^2)\Big[(1+\sqrt{s})^2-\frac{4}{n+3}\Big(\frac{1+s^2}{1+s}+\frac{n+3}{n+1}\frac{s}{1+s}+ \\ \qquad +\frac{n+4}{n+2}\sqrt{s}\Big)\Big], \quad 0\leqslant I\leqslant I_R, \\ 1-c^4-4\theta c(1-c^2+\theta c\ln c)-\frac{4I_R/I}{n+3}\big[c^2(\theta-c)(\sqrt{c}-\sqrt{\theta})^2(1+ \\ +\frac{n+4}{n+2}\sqrt{\theta/c}+(1-\theta c)(1-\sqrt{\theta c})^2\Big(1+\frac{n+4}{n+2}\sqrt{\theta c}\Big)\Big], \quad I_R\leqslant I<I_*.\end{cases} \tag{9}$$

where

$$s = c\left(\frac{1+\sqrt{\theta c}}{\sqrt{\theta}+\sqrt{c}}\right)^2\left(\frac{\theta-c}{1-\theta c}\right)^{2(n-1)/n};\qquad r_2 = R_2\frac{1-\sqrt{\theta c}}{1-\sqrt{s}}\left(\frac{I}{I_R}\,\frac{1+\sqrt{s}}{1+\sqrt{\theta s}}\right)^{1/(n-1)};$$

$$Q = kI\cdot\begin{cases}(r_2/R_2)^4(1-s^2)(1-s)^2 m/(m+2), & 0\leqslant I\leqslant I_R, \\ 1-c^4-4\theta c(1-c^2+\theta c\ln c)-\frac{2I_R/I}{m+2}[(1-\theta c)^3-(\theta-c)^3], & I_R\leqslant I<I_*,\end{cases} \tag{10}$$

where

$$s = c\left(\frac{\theta-c}{1-\theta c}\right)^{(m-1)/(m+1)};\qquad r_2 = R_2(I/I_R)^{1/(2m)}\left(\frac{1-\theta c}{1-s}\right)^{(m-1)/(2m)}.$$

The obtained laws are interesting in that they allow us to take into account the singularity of the interphase effects θ at the boundary of contact of the liquid with the solid wall (or with any other flow component) and are more general for flows in capillaries of round and annular section.

For instance, from (9), (10) we can derive as particular cases formulas for the flow of: a generalized Newtonian liquid in a round capillary [8] when $s \to 0$ $(c \to 0)$; a Newtonian liquid in an annular capillary [1] when n and $m \to \infty[\theta \to (c^2-1)/(2c\ln c)]$; water in the annular space at the wall of a round tube when its central cylindrical region is occupied by air [10] when n and $m \to \infty$ and $\theta \to c$.

We note that if a relation between the parameters of the interphase characters can be established in the form θ = const $(0 \leq \mu \leq \infty)$, the flow of liquid in formulas (9) and (10) will be expressed only as a function of the pressure gradient I and the internal rheological characteristic of the liquid n, m; and, on the other hand, on the basis of (9), (10) from experimentally determined flow laws we can conduct a search for the values of n, m, and θ.

A flow of generalized Bingham liquids under the action of external forces $I \geq 0$ exceeding the forces of cohesion of the liquid particles with the solid wall begins immediately throughout the volume of the capillary and, when $I \geq I_0$, is due to the gradual lamination of the moving mass within the expanding regions $R_1 \leq r \leq r_1$ and $r_2 \leq r \leq R_2$. In this case the intermediate region $r_1 \leq r \leq r_2$ of liquid moves as a single whole. i.e., like a solid body. and when I attains the value I_* the liquid in the annular gap is completely laminated, and when $I \geq I_*$ the flow becomes viscous [9].

In this case the velocity $v_j(r)$ of a liquid particle in each zone is given by the formula

$$v_j(r) = -\frac{\rho g I}{4\eta}\left(r^2-R_j^2-2r_j^2\ln\frac{r}{R_j}\right)-(-1)^j\Big[r_jN_j(r_j)\ln\frac{z}{R_j}- \\ -\int_{R_j}^{r}N_j(r)\,dr\Big]\quad (j=1;2),\quad I_0\leqslant I<I_*, \tag{11}$$

which, like (3), is obtained by integration of (1) with boundary conditions

$$\text{a)}\quad v_j(R_j)=0,\quad \text{b)}\quad \left.\frac{dv_j(r)}{dr}\right|_{r=r_j}=0,\quad \text{c)}\quad v_1(r_1)=v_2(r_2).$$

We note that when $N_j(r) = \tau_0/\eta$ = const, we arrive. from (11), at the corresponding equations previously obtained for a flow of viscoplastic liquid in an annular space [5].

For this flow model, taking $N_j(r)$ in the form of the first representation in (6), we have $M_j = M$, $r_0 = (R_2r_1 - R_1r_2)/(R_2 - r_2 + r_1 - R_1)$ and

$$N_j(r_j)=M\left(\frac{r_2-r_1}{R_2-R_1}\right)^n,\quad -\infty\leqslant n\leqslant 1.$$

Now, writing the equilibrium condition

$$\pi\left(r_2^2-r_1^2\right)\rho g I=2\pi\left(r_2+r_1\right)\eta M\left(\frac{r_2-r_1}{R_2-R_1}\right)^n$$

in accordance with the established flow regime for the initial and effective pressure gradients, we obtain

$$I_0=\frac{2\eta M}{\rho g\left(R_2-R_1\right)},\quad I=I_0\left(\frac{r_2-r_1}{R_2-R_1}\right)^{n-1}.$$

For determination of r_j (and then r_0) the usual procedure [5] is to consider the latter relation along with the boundary condition c).

In accordance with the above arguments, the flow Q of a generalized Bingham liquid in an annular capillary is given in the form

$$Q=2\pi\sum_{j=1}^{2}(-1)^j\int_{r_j}^{R_j} r v_j(r)\,dr+\pi\left(r_2^2-r_1^2\right)v_1(r_1)=$$
$$=\frac{\pi\rho g I}{8\eta}\left\{\sum_{j=1}^{2}\left[(-1)^j\left(R_j^2-r_j^2\right)^2+2r_j\left(r_2-r_1\right)\left(R_j^2-r_j^2\right)\right]+\right.$$
$$\left.+\frac{4\left(r_2-r_1\right)\left(R_2-r_0\right)^3}{(n+1)(n+2)(n+3)}\left[F_n\left(r_1,r_2,I\right)-F_n\left(R_1,R_2,I_0\right)\left(I_0/I\right)^{n/(n-1)}\right]\right\},$$
$$I_0\leqslant I<I_*,\tag{12}$$

where

$$F_n\left(r_1,r_2,I\right)=(n+2)(n+3)\frac{r_2^2-\lambda r_1^2}{\left(R_2-r_0\right)^2}\left(\frac{I}{I_0}\right)^{1/(n-1)}-$$
$$-2(n+3)\frac{r_2+\lambda^2 r_1}{R_2-r_0}\left(\frac{I}{I_0}\right)^{2/(n-1)}+2\left(1+\lambda^3\right)\left(\frac{I}{I_0}\right)^{3/(n-1)},$$

and we obtain $F_n(R_1, R_2, I_0)$ from this by replacing r_1, r_2, and I, respectively, by R_1, R_2, and I_0; $\lambda = (R_1 - r_0)/(R_2 - r_0)$.

After simple algebra we obtain from (12), with n = 0, the formula for a flow of viscoplastic fluid in the annular space between two coaxial cylindrical tubes [5]

$$Q=\frac{\pi\rho g I}{24\eta}\left\{3\left[\left(R_2^2-r_2^2\right)^2-\left(R_1^2-r_1^2\right)^2\right]+\right.$$
$$\left.+2\left(r_2-r_1\right)\left[3\left(r_1R_1^2+r_2R_2^2\right)-2\left(R_1^3+R_2^3\right)-r_1^3-r_2^3\right]\right\}.$$

When $r_1 \to 0$ ($R_1 \to 0$) we obtain from (12) the formula for the flow of generalized Bingham liquid in a round capillary [8].

The above flow models for complex liquids with a choice of defined functions $N_j(r)$ can be used for the description and explanation of anomalous filtration processes in inhomogeneous porous materials.

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CONCENTRIC IMPACT OF POINTED BODIES

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In [2] the concentric press described in [1] was analyzed for the limiting case of a sphere composed of a set of narrow pyramids which occupy the sphere not continuously but with a certain porosity $K > 1$ (K is the ratio of the volume of the sphere to the total volume of the pyramids). Whereas [2] was concerned with the static action of the press, the present article deals with the dynamic process of compression in which the pyramids approach each other at a certain speed. This question arose as a natural extension of the work described in [2]. As before, the entire effect is self-similar, the compression of the material at the center of the device is infinitely great and lasts a finite time (until externally relieved). For the parts in the center not to be destroyed, it is sufficient to assume slight linear hardening of the press material under pressure; experiments [3] show that under pressure the strength increases considerably.

Diagrams showing the device at the initial moment and at a later stage are presented in Fig. 1a, b. The pyramids approach the center at the rate u_0. In the center there is formed a spherical zone of continuous compression whose boundary moves outwards at the rate v; behind it a shock wave spreads out from the center at velocity w. We note that the porosity $K = (\beta/\alpha)^2$, where α is the angle at the vertex of the uncompressed pyramid, and β is the angle at the vertex of the compressed pyramid.

Figure 2 shows the path of a lateral particle of the pyramid up to the closing of the gap. Clearly, $-u_0\alpha = v(\beta - \alpha)$ ($u_0 < 0$), whence $u_0/v = -(\sqrt{K} - 1)$, which for low porosity ($K - 1 = \varepsilon \ll 1$) gives $u_0/v = -\varepsilon/2$.

A qualitative picture of the motion is given in Fig. 3. Until the pyramids close up, the material moves at a constant rate (from q to r_0); this is followed by a smooth deceleration along the path from r_0 to r_1. At the shock wave the velocity decreases abruptly but re-

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